CLASS TEST - 07

Batch : S3 (Foundation) SOLUTION **Date : 17/09/2023**

1. **[C] Let v_1 and v_2 be the initial speeds of first second runners respectively. Let be time by them when the first runner has completed 50 m. During this time, the second runner has covered a distance = 50 - 1 = 49 m.**

So, $$t = \frac{50}{v_1} = \frac{49}{v_2} \qquad \text{...(i)}$$

Suppose, the second runner increases his speed to v_3 so that he covers the remaining distance $(= 51m)$ in time t.

So, $$t = \frac{51}{v_3} = \frac{49}{v_2}$$

or $$v_3 = \frac{51}{49} v_2$$

or $$v_3 = \left(1 + \frac{2}{49}\right) v_2$$

or $$\frac{v_3}{v_2} - 1 = \frac{2}{49}$$

or $$\frac{v_3 - v_2}{v_2} = \frac{2}{49}$$

or $$\%\ increase = \frac{2}{49} \times 100 = 4.1\%$$

2. **[B] Given , acceleration** $a = (6t+5) m/s^2$

$$\Rightarrow \qquad a = \frac{dv}{dt} = (6t+5),$$

$$dv = (6t+5)dt$$

Integrating it, we have $\int_0^v dv = \int_0^4 (6t+5)dt$

$$v = 3t^2 + 5t + c$$

where c is constant of integration

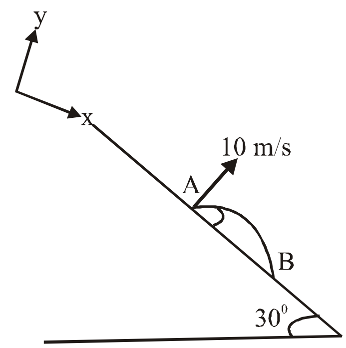
Where $t = 0, v = 0 \ so\ c = 0$

$$\therefore \qquad v = 3t^2 + 5t$$

$$\Rightarrow \qquad ds = (3t^2 + 5t)dt \qquad \left(as\ v = \frac{ds}{dt}\right)$$

3. **[D] From graph, what $t = 0$, the particle is released from rest at A, hence, $v = 0$.**
At B, at the graph is parallel to time axis, hence velocity is constant there. Thus acceleration a is zero.
At C, the graph change slope, where velocity and acceleration vanish.
Average velocity for motion between A and D is negative because the value of x is decreasing with time t. The slope of graph (which represents speed) is more at D than at E. Therefore, speed at D is more than at E.

4. **[B] At B,** $S_y = 0$



$$\therefore \qquad U_y t + \frac{1}{2} a_y t^2 = 0$$

or $$t = -\frac{2U_y}{a_y} = \frac{-2(10)}{-10 \times \sqrt{3}/2} = \frac{4}{\sqrt{3}} 5$$

Now, $$AB = R = \frac{1}{2} a_x t^2$$

$$= \frac{1}{2}\left(10 \times \frac{1}{2}\right)\left(\frac{16}{3}\right) = 13.33m$$

5. **[B]** $h = (u \sin\theta)t - \frac{1}{2} g t^2$

$$d(u\cos\theta)t \ \ or\ t = \frac{d}{u\cos\theta}$$

$$h = u\sin\theta \cdot \frac{d}{u\cos\theta} - \frac{1}{2} g . \frac{d^2}{u^2\cos^2\theta}$$

$$u = \frac{d}{\cos\theta}\sqrt{\frac{g}{2(d\ \tan\theta - h)}}$$

6. **[D]** $R = \frac{u^2 \sin 2\theta}{g} = \frac{2u_x u_y}{g}$

$\therefore$ **Range** $\propto$ **horizontal initial velocity component** (v_x)
In path 4 range is maximum of football has maximum horizontal velocity component in this path.

7. **[B] Let v be the velocity acquired by the body at B which will be moving making an angle $45°$ with the horizontal direction. As the body just crosses the well so** $\frac{v^2}{g} = 40$

or $$v^2 = 40g = 40 \times 10 = 400$$

or $$v = 20ms^{-1}$$

Taking motion of the body from A to B along the inclined plane, we have

$$u = v_{0,} a = -g\sin 45^\circ = -\frac{10}{\sqrt{2}} ms^{-2}$$

$$s = 20m, v = 20ms^{-1}$$

As $\quad v^2 = u^2 + 2as$

$$\therefore \quad 400 = v_0^2 + 2\left(-\frac{10}{\sqrt{2}}\right)\times 20\sqrt{2}$$

or $\quad v_0^2 = 400 + 400 = 800$

or $\quad v = 20\sqrt{2}ms^{-1}$

8. [B] Here the tangential acceleration also exists whichrequireds power.

Given that, $\quad a_c = k^2 \times rt^2$

and $\quad a_c = \frac{v^2}{t}$

$$\therefore \quad \frac{v^2}{r} = k^2 rt^2$$

or $\quad v^2 = k^2 r^2 t^2 \quad or \quad v = krt$

Tangential acceleration $a = \frac{dv}{dt} = kr$

Now force F = m. a = mkr

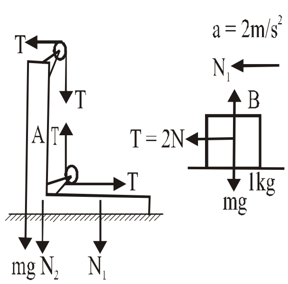
So power $P = F \times v = mkr \times krt = mk^2 r^2 t$

9. [B] Here , $T = \frac{2\pi r}{4v} = \frac{\pi r}{2v}$

Change in velocity is going from A to B $= v\sqrt{2}$

Average acceleration $= \frac{v\sqrt{2}}{\pi r/2v} = \frac{2\sqrt{2}v^2}{\pi r}$

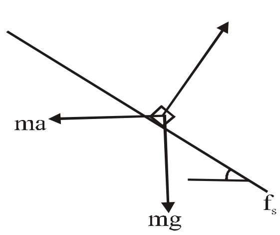
10. [C]



$$\therefore \quad a = 0 \therefore \quad Relative \quad acceleration = 2m/s^2$$

11. [C] $a_m = 0 \Rightarrow$ spring will extend.

12. [D] mg $\sin\theta + f_s = ma \cos\theta$



$$f_s = m(a\cos\theta - g\sin\theta) \le \mu(mg\cos\theta + ma\sin\theta)$$

$$\therefore \mu \ge \frac{a - g\tan\theta}{g + a\tan\theta} = \frac{1}{\sqrt{3}} = 0.577$$

$$\frac{9}{10} > 0.577$$

13. [D] $k_{eq} = \frac{100\times150}{250} = 60N/m$

$$F = k_{eq}x = 60\times\frac{2.5}{100} = \frac{3}{2}N$$

For the left spring $x_1 = \frac{3}{2(100)}$

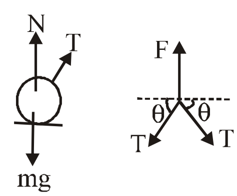
For right spring $x_2 = \frac{3}{2(150)}$

$$So, \frac{\frac{1}{2}(100)\left(\frac{3}{2}\right)^2\left(\frac{1}{100}\right)^2}{\frac{1}{2}(150)\left(\frac{3}{2}\right)^2\left(\frac{1}{150}\right)^2} = \frac{150}{100} = \frac{3}{2}$$

14. [B] $F = 2T\sin\theta$ ....(1)

$T\sin\theta + N = mg$ .....(2)

$$\therefore \quad F_{max} = 2mg$$



15. [D] $\frac{dU}{dx} = 6x - 6x^2 = 0 \Rightarrow 6x(1-x) = 0; x = 0,1$

$$\frac{d^2U}{dx^2} = 6 - 12x$$

At $x = 0, \frac{d^2U}{dx^2} > 0 \Rightarrow$ stablr equilibrium

At $x = 1, \frac{d^2U}{dx^2} < 0 \Rightarrow$ unstable equilibrium

16. [B] Applying work energy theorem; $W_{all} = \Delta KE$

Since man finally comes to rest $\Rightarrow W_{all} = 0$

Work is done by a force only, when point of application of force moves in the direction of force applied.

17 A- R, B- PSTC, C- OR, D- PS

18.

19. **[D]** $a_t = \frac{av}{dt} = 2$

$$a_c = \frac{v^2}{R} = \frac{4}{1} = 4$$

$$a = \sqrt{a_t^2 + a_c^2} = \sqrt{20} = 2\sqrt{5} m/s^2$$

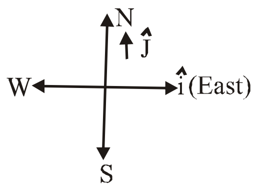
20. [**C] Relative velocity between A and B remains the same and is along the line joining the two.**

$\therefore$ **They would collide but somewhere east of line AB**

21. **[A]**

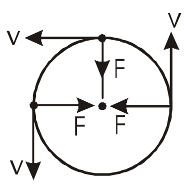
$$\vec{n}_{s,w} = (12.50)(-\vec{i})$$

$$\vec{n}_{o,w} = \vec{V}_{o,G} - \vec{V}_{w,g}$$

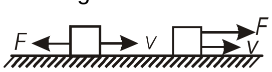


22. [C] For circular or elliptical path, direction of force should be variable.

For rectilinear path, the direction of force should be either in the direction of instantaneous velocity or opposite to instantaneous velocity.



For parabolic path the force should be constant in respect to magnitude and direction. But the only magnitude of given force is constant.



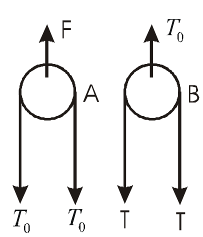
Hence, path can never be parabolic.

23. [A] Let T_0 = tension in the string passing over A

T = tension in the string passing over B

$$2T_0 = F \text{ and } 2T = T_0$$

$$\Rightarrow \quad T = \frac{F}{4}$$



$$\therefore \quad T = \frac{F}{4} = 75N$$

Weights of blocks are; mg = 50 N

mg = 100 N

As T < Mg, and T > mg, M will remain stationary on the floor, whereas m will move.

Acceleration of m,

$$a = \frac{T - mg}{m} = \frac{75 - 50}{5} = 5m/s^2$$

24. **[B]** For equilibrium,

$kx_1 \sin 60 = k x_3 \sin 60$

$\therefore \quad x_1 = x_3$

and $\quad kx_1 \cos 60 + k_0x_2 + kx_3 \cos 60 = mg$

or $\quad kx_1 + k_0x_2 = mg = 100$ N

$\therefore \quad kx_1 = 100 - kx_2 = 100 - 1000 \times \frac{5}{100} = 50$N

After breaking

$$a_x = \frac{kx_1 \sin 60}{m} \text{ and } a_y = \frac{mg - kx_1\cos 60 - k_0x_2}{m}$$

$$= \frac{50\sqrt{3}}{2\times 10} = \frac{5}{2}\sqrt{3} m/s^2 \text{ and } a_y = \frac{100-25-50}{10} = \frac{25}{10} = 2.5 m/s^2$$

$$\therefore \quad a = \sqrt{a_x^2 + a_y^2} = \sqrt{\frac{75}{4} + 6.25}$$

$$a = \sqrt{25} = 5 m/s^2$$

25. **[A]**

$$W = \vec{F}.\vec{S}$$

where $\vec{S}$ = displacement = $\vec{r_2} + (-\vec{r_1})$

$$\vec{S} = (i + 3j - 3k)$$

$$W = \left(2\hat{i} + \hat{j} - \hat{k}\right).\left(\hat{i} + 3\hat{j} - 3\hat{k}\right)$$

$$W = 2 + 3 + 3 = 8$$

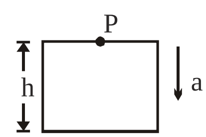
26. **[C]**

Acceleration of particle in the lift frame = (g – a)

At t = 0, velocity of particle and lift are zero.

Hence in lift frame

$$h = \frac{1}{2}(g - a)t^2$$



$$t = \sqrt{\frac{2\times 3}{10 - 4}} = 1 \text{ sec}$$

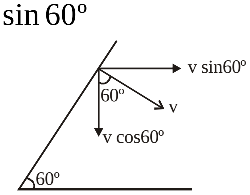
Distance travelled by particle in ground frame

$$S = \frac{1}{2}gt^2 = \frac{1}{2}\times 10 \times 1^2 = 5 \text{ m}$$

27. **[C]**

Let v be the velocity at the time of collision.

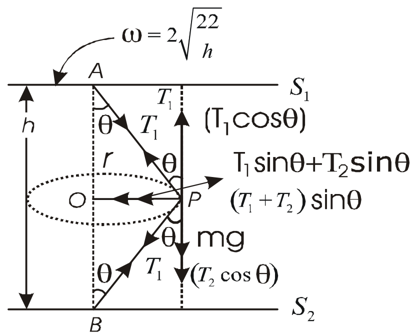
Then, $\quad u\sqrt{2}\cos 45^o = v \sin 60^o$



$$\left(u\sqrt{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{3}\,v}{2}$$

$\therefore \quad \boxed{v = \frac{2}{\sqrt{3}} u}$

28. **[B]**



From the figure it is clear that object P is tied with identical string, so Here (T_1+T_2)

$\angle OAP = \angle PBO = ¯$ sin ¯ will provide required centripetal force.

So, $(T_1 + T_2)$ sin ¯ = mr μ^2 ...(i)

and T_1cos ¯ = mg + T_2 cos ¯

cos¯ $(T_1 - T_2)$ = mg ...(ii)

By Eqs. (i) and (ii), we get

$$\frac{(T_1+T_2)\sin¯}{(T_1-T_2)\cos¯} = \frac{r\mu^2}{g}$$

$$\frac{(T_1+T_2)}{(T_1-T_2)}\tan¯ = \frac{\mu^2}{g}\left[\tan¯ \times \frac{h}{2}\right]$$

$$\frac{(T_1+T_2)}{(T_1-T_2)} = \frac{\mu^2 h}{2g} = \left(4\times\frac{2g}{h}\right)\times\frac{h}{2g}\left[\therefore \tan¯ = \frac{r}{(h/2)}\right]$$

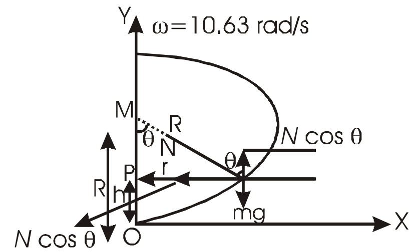
$$\left(As,\ \mu = \sqrt[2]{2g/h}\right)$$

$(T_1 + T_2) = 4(T_1 - T_2) \Rightarrow T_1 + T_2 = 4T_1 - 4T_2$

$3T_1 = 5T_2 \quad \Rightarrow \quad \frac{T_1}{T_2} = \frac{5}{3}$

$$\frac{V}{\mu g} = t$$

29. [A] Problem solving strategy The normal reaction will balance the weight of the bead and provide the required centripetal force.



N cosq will balance mg.

[N = normal reaction force]

N cos¯ = mg ...(i)

N sin ¯ will provide the required centripetal force.

N sin¯ = mr μ^2 [As, r = Rsinq]

N sin¯ = m(Rsinq) μ^2 ...(ii)

From Eqs. (ii) and (i) by dividing, we get

$$\tan¯ = \frac{R\sin¯\mu^2}{g}$$

$$\frac{\sin¯}{\cos¯} = \frac{R\sin¯ a^2}{g}$$

$$\frac{1}{\cos¯} = \frac{R\mu^2}{g}$$

$$\cos¯ = \frac{g}{R\mu^2} = \frac{100}{0.1\times(10.63)^2}$$

$$\cos¯ = 0.885$$

$$\Rightarrow \quad h = OM - MP = 0.1 - 0.1\cos¯$$

$$[MP = R\cos¯, OM = R]$$

$$= 0.1[1 - 0.885] = 0.0115$$

30. **[B]**

Average pseudo force = $\frac{\textbf{change in momentum in non - inertial frame}}{\textbf{time elapsed}}$

$= \frac{\mathbf{3\pi}}{\mathbf{10}}$**N.**