Location of centre of mass

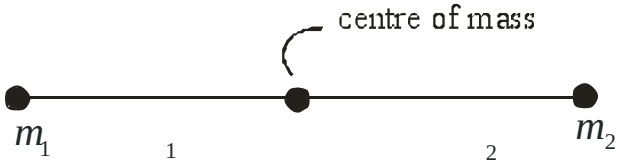
SHORT NOTES & ASSIGNMENT

(1) co-ordinates of centre of mass are $x_{cm} = \frac{m_1x_1 + m_2x_2 + ...m_nx_n}{M}$

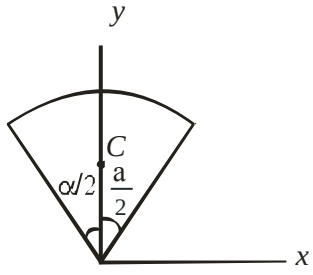
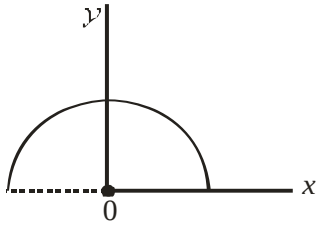
$y_{cm} = \frac{m_1y_1 + m_2y_2 + ...m_ny_n}{M} = \frac{\Sigma m_i y_i}{\Sigma m_i}$ And $Z_{cm} = \frac{m_1z_1 + m_2z_2 + ...m_nz_n}{M} = \frac{\Sigma m_i z_i}{\Sigma m_i}$

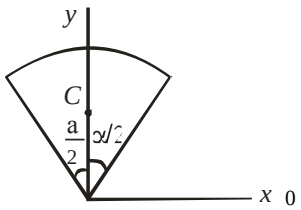
If the position vectors of particles are $\vec{\mathbf{r}}_1, \vec{\mathbf{r}}_2, ...\vec{\mathbf{r}}_n$ respectively then the position vector of centre of mass is defined as $\vec{\mathbf{r}}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{\mathbf{r}}_2 + ... + m_n\vec{\mathbf{r}}_n}{M} \cdot = \frac{\Sigma m_i \vec{r}_i}{\Sigma m_i}.$

(2) Centre of mass divides the distance between two particles in the inverse ratio of masses. i.e. $\frac{\ell_1}{\ell_2} = \frac{m_2}{m_1}$



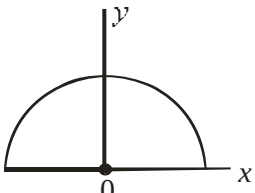
(3) Centre of mass of some common uniform bodies

(a) Name	Shape	x_{cm}	y_{cm}
(i) Circular arc		0	$\frac{2R}{\sim}\sin\frac{\sim}{2}$
(ii) Semicircular arc,		0	$\frac{2R}{®}$
(iii) Sector of a circular plate			



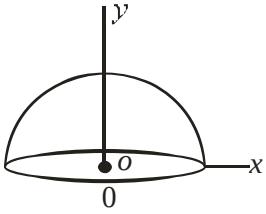
$\frac{4R}{3\sim}\sin\frac{\sim}{2}$

(iv) Semicircular plate



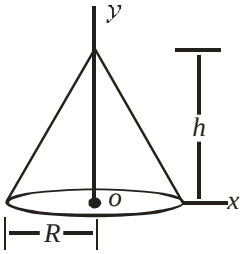
here a =p 0 $\frac{4R}{3®}$

(v) hollow hemisphere



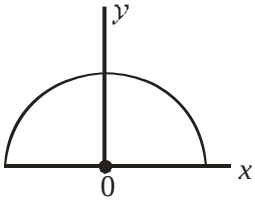
0 $\frac{R}{2}$

(vi) Right circular hollow cone



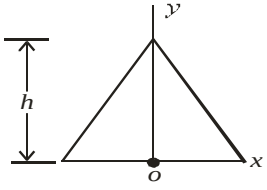
0 $\frac{h}{3}$

(vii) Solid hemisphere



0 $\frac{3R}{8}$

(viii) Right circular solid cone



0 $\frac{h}{4}$

(4) The centre of mass may lie outside the body (e.g. ring).

(5) There need not be a mass at the centre of mass.

(6) If acceleration due to gravity is constant, centre of mass coincides with centre of gravity.

(7) If a body is performing translational motion without rotating, net torque on body about centre of mass is zero.

(8) The centre of gravity can be different from the centre of mass only when acceleration due to gravity is not parallel and uniform.

(9) If a body is suspended by a thread from a point on its surface, the line of suspension of the thread passes through the centre of gravity.

(10) Gravitational potential energy of a system of particles near the surface of earth may be calculated as if the entire mass of system were located at the centre of mass.

(11) If equal masses are placed at the corners of a regular polygon, centre of mass of system coincides with the centre of polygon.

(12) If we know the centre of mass of a number of different systems and their masses, we can get the combined centre of mass by treating the different system as point particles placed at their respective centre of mass.

(13) The centre of mass of a homogeneous, symmetric body must be at the geometric centre. For example, centre of mass of uniform thin rod, uniform circular ring, uniform circular disc, uniform triangular plate, uniform rectangular plate, uniform sphere, uniform cube etc are at their respective geometrical centre.

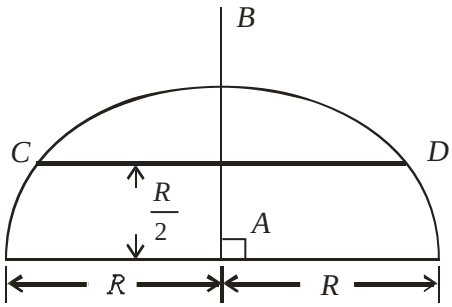
(14) The centre of mass of uniform triangular plate coincides with that of three equal masses placed at its vertices.

(15) To determine the centre of mass of body with a cutout portion firstly we consider whole portion of plate including cutting portion. If we consider the cutting portion with a negative mass equal in magnitude to the mass of cutting portion. Then combined centre of mass of these two considered systems is the centre of mass of remaining part of the body.

EXERCISE

Conceptual questions

1. Consider earth and moon as a system. Locate the region in which centre of mass of the Earth-moon system?
2. An object has its centre of mass at the origin. May the x-co-ordinates of the particles be all non negative?
3. Is the centre of mass of Himalaya higher or lower than the centre of gravity?
4. A child has to divide a uniform semicircular biscuit in two equally massive parts (shown in figure) by cutting either along line AB or along line CD. Which cut is correct?



5. A solid uniform hemisphere of radius R is cut by a plane at $\frac{R}{2}$ from curvature centre parallel to base. Which part has more mass?
6. If density of a rod continuously increases from one end to other end. Is the centre of mass of rod certainly not at its geometrical centre?

Only One Option Correct Type

7. The distance between the centre of carbon atom and the centre of oxygen atom in carbon monoxide molecule is 1.1 A^o. Locate the position of centre of mass of the molecule from centre of carbon atom.

(A) $1A^o$ (B) $0.5A^o$ (C) $0.63A^o$ (D) $0.74A^o$

8. Particle of masses 1g, 2g, 3g, -----n g are placed on the position x-axis at distance 1 cm, 2 cm, 3 cm, ----- n cm from the origin. Find x-co-ordinate of position of centre of mass of the system.

(A) $\frac{n(n+1)}{2}$ (B) $n^2(n+1)$ (C) $\frac{2n+1}{3}$ (D) $\frac{2n-1}{3}$

9. Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved, so as to keep the centre of mass at the same position?

(A) $\frac{m_2}{m_1}d$ (B) $\frac{m_1}{m_1+m_2}d$ (C) $\frac{m_1}{m_2}d$ (D) d

10. Four point mass $m_1 = 1g,\ m_2 = 2g,\ m_3 = 3g$ and $m_4 = 4g$ are placed at respective corners of a square of side 10 cm. Find distance of centre of the system from the particle of 1g.

(A) 1.07 cm (B) 5 cm (C) 8.6 cm (D) 6.7cm

11. Two particles of masses 1 g and 3g are at A(1, 2, 1)m and B(-3, -2, 1) m respectively. A third particle of mass 4 g is placed at point P such that the centre of mass of the system is at the origin. Find the position vector of the particle P.

(A) $2\hat{i}+\hat{j}-\hat{k}$ (B) $\hat{i}+\hat{j}-\hat{k}$ (C) $3\hat{i}+4\hat{j}+5\hat{k}$ (D) $3\hat{i}-\hat{j}+\hat{k}$

12. Three particles of masses 1g, 2g and 3g are at (1, 2, 3)m (3, 4, 5)m and (-2, -3, 4) respectively.
Where should a particle of mass 4g be placed so that the centre of mass of combined system will be at (2, 3, 4)m.

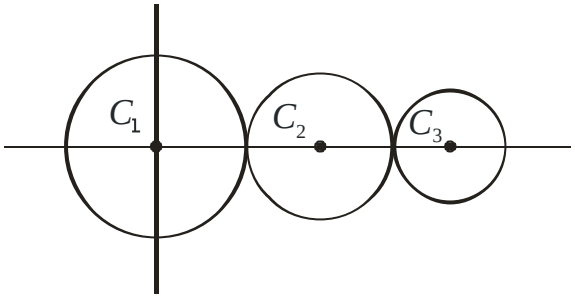
(A) $\frac{19}{4}\hat{i}+\frac{29}{4}\hat{j}+\frac{15}{4}\hat{k}$ (B) $4\hat{i}+3\hat{j}-2\hat{k}$

(C) $19\hat{i}+29\hat{j}+30\hat{k}$ (D) none

13. A thin uniform rod of mass M and length 2R is attached to a point on the circumference of a uniform circular disc of mass 2M and radius R. Find the distance of centre of mass of disc plus rod system from the centre of disc.

(A) $\frac{R}{2}$ (B) $\frac{R}{3}$ (C) $\frac{4R}{3}$ (D) $\frac{2R}{3}$

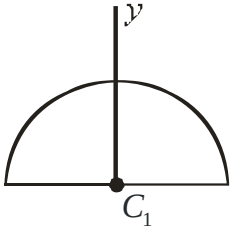
14. Three identical uniform discs, each of mass 2 kg and radius 10 cm are kept touching each other, with their centre on a straight line (shown in figure).



Find distance of centre of mass of the system from C_1.

(A) 10 cm (B) 20 cm (C) 30 cm (D) 40 cm

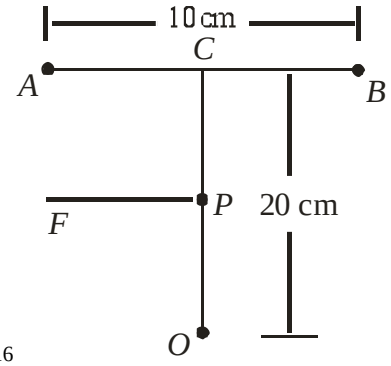
15. Locate centre of mass of a uniform semicircular loop of radius R (shown in figure)



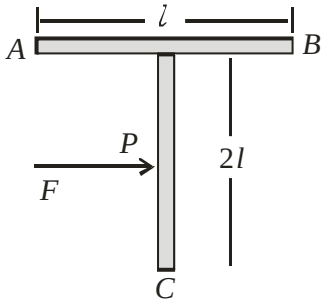
(A) $\frac{2R}{®}$ above C_1 (B) $\frac{2R}{2+®}$ above C_1

(C) $\frac{R}{2+®}$ above C_1 (D) $\frac{3R}{2+®}$ above C_1

16. A *T* shaped light wire frame (shown in figure) is lying on a smooth horizontal floor.



PROBLEM 16



PROBLEM 17

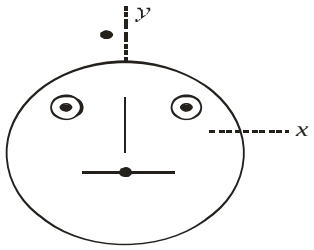
Equal masses are attached at its ends A, B and O. Constant force F is applied at point P parallel to AB, such that the system has only the translational motion without rotation. Find the distance of point P from point C.

(A) $\frac{10}{3}cm$ (B) $\frac{5}{3}cm$ (C) $\frac{20}{3}cm$ (D) 10 cm

17. A *T* shaped object with dimensions shown in the figure, is lying on a smooth floor. A force F is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C.

(A) $\frac{2}{3}l$ (B) $\frac{3}{2}l$ (C) $\frac{4}{3}l$ (D) l

18. Look at the drawing given in the figure, which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is m. The mass of the ink used to draw the outer circle is 6 m. The coordinates of the centres of the different parts are: outer circle (0, 0) and inner circle (-a, a), right inner circle (a, a), vertical line (0, 0) and horizontal line (0, -a). They y-coordinate of the centre of mass of the ink in this drawing is

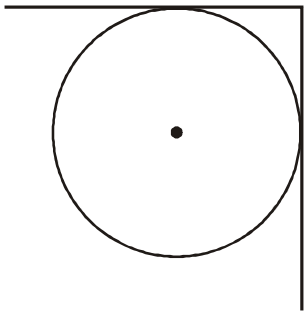


(A) $\frac{a}{10}$ (B) $\frac{a}{8}$ (C) $\frac{a}{12}$ (D) $\frac{a}{3}$

19. Distance of the centre of mass of a solid uniform cone from its vertex is z_0 . If the radius of its base is R and its height is h, then z_0 is equal to

(A) $\frac{3h}{4}$ (B) $\frac{h^2}{4R}$ (C) $\frac{5h}{8}$ (D) $\frac{3h^2}{8R}$

20. A uniform thin rod of length 2m bent at right angle at its middle point is placed onto a horizontal fixed uniform disc having radius 1 m (shown in figure). Find the distance of centre of mass of the system (thin rod + disc) from the bending point of the rod.



The numerical value of surface mass density of disc is same as numerical value of linear mass density of the thin rod.

(A) $\left(\frac{3® +1}{4+2®}, \frac{3® +1}{4+2®}\right)$ (B) $\left(\frac{2® +1}{4+2®}, \frac{2® +1}{4+2®}\right)$

(C) $\left(\frac{® +1}{4+2®}, \frac{® +1}{4+2®}\right)$ (D) $\left(\frac{3® +1}{1+2®}, \frac{3® +1}{1+2®}\right)$

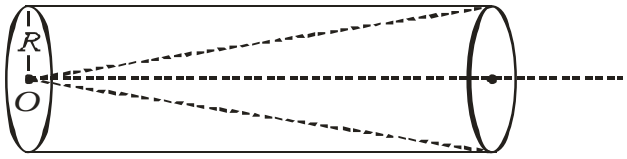
21. A circular disc of radius R is removed from a bigger circular disc of radius 2R, such that the circumference of the discs coincide. The centre of mass of the new disc is $R\sim$ from the centre of the bigger disc. The value of $\sim$ is

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{1}{6}$ (D) $\frac{1}{4}$

22. A uniform square plate of side length ℓ has its centre at O. A square shape portion of side length $\frac{\ell}{4}$ is removed whose centre is at distance b from the point O. The cavity is filled with a material of surface mass density twice of that of the plate. Find distance of centre of mass of the composite plate from the point O.

(A) B (B) $\frac{b}{2}$ (C) $\frac{b}{16}$ (D) $\frac{b}{17}$

23. A conical portion is cut from a uniform cylinder of radius R and length ℓ (shown in figure)



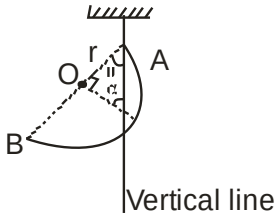
Find distance of centre of mass of remaining portion from point O.

(A) $\frac{1}{4}cm$ (B) $\frac{3}{4}cm$ (C) $\frac{3}{8}cm$ (D) $\frac{3}{16}cm$

24. An empty thin uniform cylindrical vessel of mass 200 gm, radius 6 cm has its centre of mass 2 cm above its base. To what height should it be filled with water of density 1 gm/cc so as to make it as stable as possible? (Take $\pi = 3$

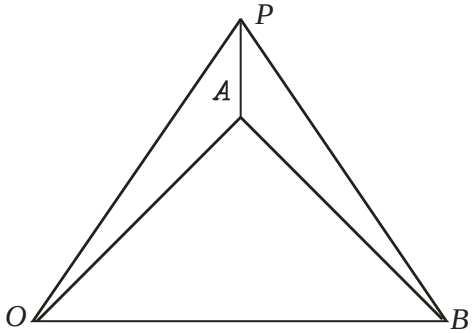
(A) 2 cm (B) 3 cm (C) 5.14 cm (D) 9 cm

25. A thin uniform semicircular wire is suspended through a string attached at point A of the wire as shown in figure. Find angle between the diameter and the vertical line.



(A) $\sin^{-1}\frac{2}{\pi}$ (B) $\tan^{-1}\frac{2}{\pi}$ (C) $\cos^{-1}\frac{2}{\pi}$ (D) None

26. Light strings of length ℓ_1, ℓ_2 and ℓ_3 are attached to the corners of a uniform triangular plate of mass m. The other ends of the strings are attached to a common point as shown in figure. If tension in strings are T_1, T_2 and T_3. Find the value of $T_1 : T_2 : T_3$.



(A) $\ell_1 : \ell_2 : \ell_3$ (B) $\ell_3 : \ell_2 : \ell_1$ (C) $\ell_1^2 : \ell_2^2 : \ell_3^2$ (D) $\ell_1^3 : \ell_2^3 : \ell_3^3$

One or More than One Options Correct Type

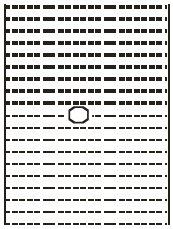
27. Four point masses are placed at the corners of a quadrilateral, the centre of mass
(A) always lies inside the quadrilateral
(B) may lie inside quadrilateral
(C) may lie outside quadrilateral
(D) always lies outside quadrilateral.

28. A radish is cut at the location of its centre of mass.
Then,

(A) the two pieces will have same mass

(B) the thicker piece will have larger mass
(C) the thin piece will have larger mass
(D) the two pieces will have different axial length.

29. A cylindrical vessel is filled with water of density 1 gm/cc. A steel ball of radius $\left(\frac{3}{4®}\right)^{\frac{1}{3}}$ cm and density $2g/cm^3$ is released into the water. Initially, the centre of ball is at height $\left[10+\left(\frac{3}{4®}\right)^{\frac{1}{3}} cm\right]$ above the bottom of the vessel. Find the distance through which the centre of mass of the system (water + ball) shifts when ball reaches at the bottom of vessel.

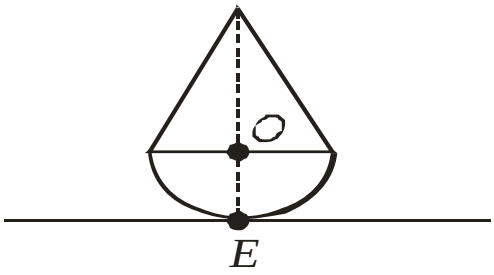


(A) The centre of mass shifted through $\frac{10}{3}$ cm in upward direction

(B) The centre of mass shifted $\frac{10}{3}$ cm in downward direction

(C) Centre of mass must shift.
(D) Centre of mass shifts in horizontal direction.

30. Consider a composite body, consisting of a hemisphere and a cone having a common base, resting with the hemisphere in contact with a plane surface. (Shown in figure)



(A) If centre of gravity of composite body is at point O, the equilibrium is neutral.
(B) If centre of gravity of composite body is above point O, the equilibrium is unstable.
(C) If centre of gravity of composite body is below point O, the equilibrium is stable.
(D) none of the above.

Comprehension Type

Passage

Theorems of pappus–Guldinus are two very important theorems which deal with the surfaces and volumes of revolution. First theorem states that the area of a surface generated by rotating a uniform plane curve (e.g. A line, arc etc) for a complete rotation about a fixed axis equal to the length of the plane curve times the distance travelled by the centre of mass of the uniform plane curve in a complete revolution.
Second theorem states that the volume of a body generated by rotating a uniform thin lamina about a fixed axis is equal to the area of the thin lamina times the distance travelled by the centre of mass of the uniform lamina in a complete revolution.

31.

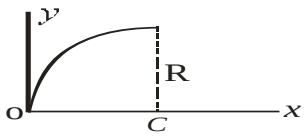
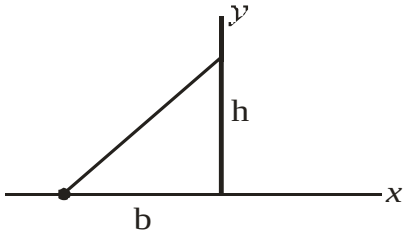


Figure shows a quarter arc of radius R.
Find y-coordinate of centre of mass of quarter arc.

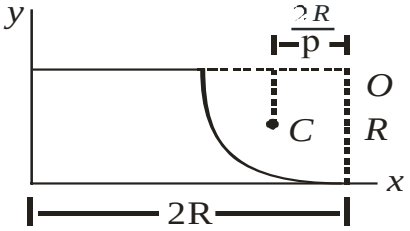
(A) $\frac{R}{®}$ (B) $\frac{2R}{®}$ (C) $\frac{3R}{®}$ (D) $\frac{4R}{®}$

32. Figure shows a uniform right-angle triangular plate of base a and height h. Find co-ordinates of centre of mass of the plate.



(A) $\frac{b}{3},\frac{h}{3}$ (B) $\frac{2b}{3},\frac{h}{3}$ (C) $\frac{b}{3},\frac{2h}{3}$ (D) $\left(\frac{b}{\sqrt{3}},\frac{h}{\sqrt{3}}\right)$

33. Find the area of the surface generated by rotating a quarter circular arc about y-axis (shown in figure).



(A) $\frac{®r^2}{4}$ (B) $\frac{®r^2}{3}$ (C) $2®R^2\left(® - 1\right)$ (D) $2®R^2$

Statement Type

Direction: This section is based on Statement I and Statement II. Select the correct answer from the codes given below.

A. Both Statement I and Statement II are correct and Statement II is the correct explanation of Statement I
B. Both Statement I and Statement II are correct and Statement II is not the correct explanation of Statement I
C. Statement I is correct but Statement II is incorrect
D. Statement II is correct but Statement I is incorrect

34. Statement I– If the density variation is symmetrical about centroid, the centre of mass and centre of volume of a body coincide.
Statement II In the case of uniform density, the centre of mass and centre of volume of a body coincide.

35. Statement I Centre of mass of a body may lie inside the body.
Statement II Centre of mass of a body must lie inside the body.

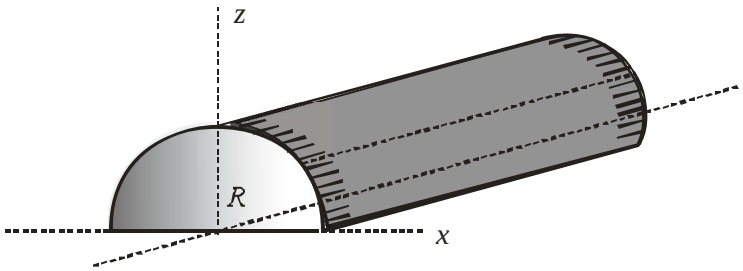
36. Statement I Centre of mass of a body is a point which can be made to lie on or outside the body by changing the co-ordinate system.
Statement II Centre of mass of a body is independent of co-ordinate system.

37. Statement I The centre of the mass of homogeneous, symmetric body must lie on its axis of symmetry.
Statement II In uniform gravitational field centre of mass and centre of gravity of a body are at the same point.

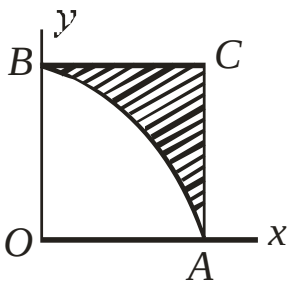
38. Statement I Two uniform sphere of same material are in constant with on another. The mass of sphere A is five times that of sphere B. The centre of mass is system is inside sphere A somewhere on the line joining the centre of A and B.
Statement II The centre of mass divides internally the line joining the two particles in inverse ratio of their masses.

One Integer Value Correct Type

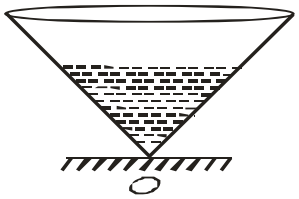
39. A uniform solid right circular cylinder of radius R and length *l* is split by a plane containing the axis of the cylinder. If the co-ordinates of the centre of mass of one split part are $0,\frac{l}{m},\frac{4R}{n®}$. Find the value of mn.



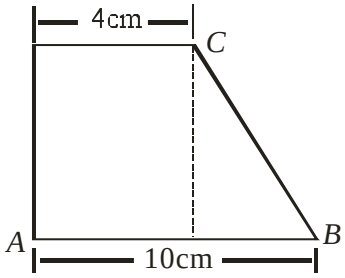
40. A quadrant of circle has be cut out from a uniform square plate of side ℓ. The distance of centre of mass of remaining portion from point O (shown in figure) is = $\frac{2\sqrt{n}\ell}{12-3\pi}$. Find the value of n.



41. A uniform conical vessel of height 15 cm and mass 100 gm is filled with water of mass 50 gm up to height 10 cm (shown in figure). If the centre of mass is $\frac{55}{n}cm$ above the point O, Find the value of n.



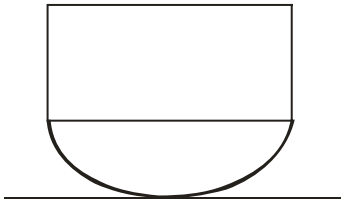
42. Figure shows a trapezoidal thin uniform plate the distance of centre of mass of the plate from AD is $\cos^{-1}\frac{2}{n\pi}$ Find value of n.



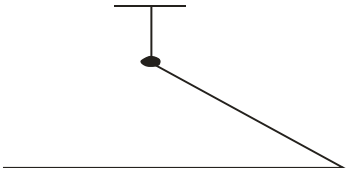
43. Suppose a thin rod of length 5m has a linear mass density $\mu = l_0 x^2$ where l_0 is constant. The rod lies along the x-axis as such its one end is at the origin. Find x-co-ordinate (in metre) of centre of mass of the rod.

44. A uniform solid hemisphere of curvature radius $R = \frac{64}{3}cm$ is cut by a plane at $\frac{R}{2}$ from curvature centre parallel to the base. Find distance (in cm) of centre of mass of lower portion from the centre of curvature.

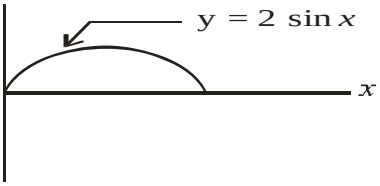
45. A right circular uniform cylinder and a uniform hemisphere having the same radius of $6\sqrt{2}cm$ joined together face to face. The density of hemisphere is same as that of the cylinder.
Find the maximum height (in cm) of the cylinder so that the composite body may rest with any point of the hemisphere in contact with a horizontal plane.



46. Two thin identical rods are joined to each other at ends (shown in figure). When the system is suspended through a string from a fixed point is formed that the lower rod is horizontal. If angle between the rods is $2\tan^{-1}\left(\frac{1}{\sqrt{n}}\right)$. Find the value of n.



47. Find ratio of x-coordinate only y-coordinate centre of mass of sinusoidal uniform plate shown in figure.



Answer Key

1. The centre of mass the system is near to earth somewhere on the line joining the centre of earth and moon.

2. Yes 3. Higher 4. Along AB 5. Base part. 6. Yes 7. (C)

8. (C) 9. (C) 10. (C) 11. (A) 12. (A) 13. (D) 14.(B) 15.(B) 16.(C)

17. (C) 18. (A) 19. (A) 20. (B) 21.(A) 22.(D) 23. (C) 24. (C) 25. (B)

26. (A) 27. (BC) 28.(BD) 29.(BC) 30.(ABC) 31.(B) 32. (A)33.(C)

34.(B) 35. (C) 36.(D) 37.(B) 38.(A)

39.[6] 40. [2] 41.(6) 42.[7] 43. [3.75]m 44. [7] 45. [6] 46. [2] 47.[2]