Time : 3 hour

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(MOTION OF CENTRE OF MASS

SHORT NOTES & ASSIGNMENT

1. Velocity of centre of mass of a system of a particle is

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_n\vec{v}_n}{m_1 + m_2 + m_3...... + m_n}$$

2. Momentum of a system of n particle is $P = M\vec{v}_{cm}$ Where

$M = m_1 + m_2 + ...m_n$

3. The acceleration of centre of mass of a system of n particle is

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + ... + m_n\vec{a}_n}{m_1 + m_2 + m_3 + ... + m_n}$$

4. The centre of mass cannot be acceleratted by internal forces of the system.

5. Net external forces on a system equals total mass of system, M multipltied by the acceleration of centre of mass, a_{cm} .

i.e. $\overrightarrow{Fext} = M\vec{a}_{cm}$.

6. The centre of mass moves like a particle of mass M equals total mass of the system under the influence of the resultant external force on the system.

7. If net external force on a system is zero and its centre of mass is intially at rest, it remains fixed even when the particles individually moves and accelerates.

In this case, if $\Delta\vec{r}_1, \Delta\vec{r}_2, \Delta\vec{r}_3....\Delta\vec{r}_n$ are displacement of particle of masses of $m_1, m_2, m_3.....m_n$ respectively.

Then, $m_1\Delta\vec{r}_1 + m_2\Delta\vec{r}_2 + ... + m_n\Delta\vec{r}_n = \vec{0}$.

If relative displacement of i^{th} particle seen from first particle is $\Delta\vec{r}_{i1}$, then the displacement of first particle is

$$\Delta\vec{r}_1 = -\frac{\sum_{i=2}^{n} m_i\Delta\vec{r}_{i1}}{\sum_{i=1}^{n} m_i}$$

8. If net external force on a system of particle is zero, the acceleration on centre of mass of system is zero. In this case the centre of mass moves with constant velocity. The displacement of centre of mass of the system is $\vec{S} = \vec{v}_{cm}t$

9. If net external force on a system of particle is non-zero, the acceleration of the centre of mass of the system is non-zero and the velocity of the centre of mass changes.

In this case

(A) If the acceleration of centre of mass is constant.

$$\vec{S}_{cm} = \vec{u}_{cm}t + \frac{1}{2}\vec{a}_{cm}t^2$$

$$\vec{v}_{cm} = \vec{u}_{cm} + \vec{a}_{cm}t$$

(B) If the acceleration of centre of mass is variable $\vec{a}_{cm} = \frac{d\vec{v}_{cm}}{dt}$

$$\vec{v}_{cm} = \frac{d\vec{s}_{cm}}{dt}$$

10. The momentum of a system in C-frame is zero.

11. Kinetic energy of a system is $T = T_{cm} + \frac{1}{2}mv_{cm}^2$

12. The mechanical energy of a system is $E = E_{cm} + \frac{1}{2}mv_{cm}^2$

13. For a system of two particles

(i) The momentum of first particle in C-frame is $\vec{P}_{1c} = \mu\,\vec{v}_{12}$

(ii) The momentum of second particle in C-frame is $\vec{P}_{2c} =$ « $\vec{v}_{21}$

(iii) $|\vec{P}_{1c}| = |\vec{P}_{2c}| =$ « $|\vec{v}_{rel}|$

(iv) The kinetic energy of the system is $T_{cm} = \frac{1}{2}$ « $|\vec{V}_{rel}|^2$.

Here, « $= \frac{m_1m_2}{m_1 + m_2}$

14. If net external force on a system is zero, the C-frame is inertial in nature.

15. If net external force on a system is non-zero, the C-frame is non-inertial in nature.

CONCEPTUAL QUESTIONS

1. Can centre of mass of a system be accelerated by internal forces of the system ?

2. Prove that the rate of change of momentum of a system is equal to product of total mass of system and the acceleration of centre of mass of the system.

3. Does the centre of mass of an isolated system move at constant velocity?

4. Ram is standing on a smooth horizontal surface facing his dog at rest. He wants to catch his dog by throwing a ball horizontally. In which direction he throws the ball ?

5. If a projectile explodes in air in a number of fragments. Comment on the path of centre of mass after explosion. (Neglect air resistance)

6. Ballet dancer creates the illusion that she floats

7. Why does a high jumper bend his body during crossing over the bar ?

8. What is momentum of a system in C-frame?

9. Is kinetic energy of system minimum in C-frame?

ONLY ONE OPTION CORRECT TYPE

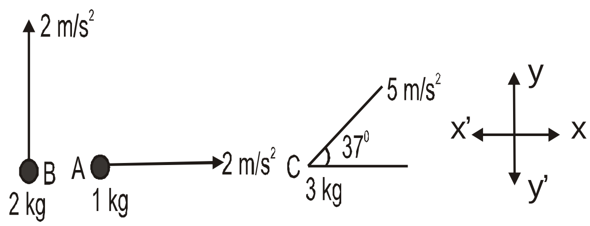
10. A thin uniform rod of mass and length 1 m is rotating with angular Velocity 2rad/s about a fixed axis passing through centre of rod and perpendicular to the length of rod. The linear momentum of rod is

(A) Zero (B) 4 kg m/s (C) 2 kg m/s (D) none

11. A system consists of four particle of masses 1 kg, 2 kg, 3 kg and 4 kg moving with velocity $\left(3\hat{i}+4\hat{j}+5\hat{k}\right)$ m/s $\left(\hat{i}-\hat{j}+\hat{k}\right)$ m/s, $\left(2\hat{i}-\hat{j}\right)$ m/s and $\left(4\hat{i}+\hat{k}\right)$ m/s respectively. The velocity of centre of mass of the system is

(A) $\left(2.7\hat{i}-0.1\hat{j}+1.1\hat{k}\right)m/s$ (B) Zero

(C) $\left(2.7\hat{i}+0.1\hat{j}+1.1\hat{k}\right)m/s$ (D) $\left(3\hat{i}-0.2\hat{j}-0.3\hat{k}\right)$

12. 

Find acceleration of centre of mass of the system at the instant shown in figure.

(A) $\left(\frac{7}{3}\hat{i}+\frac{13}{6}\hat{j}\right)m/s^2$ **(B)** $\left(7\hat{i}+13\hat{j}\right)m/s^2$

(C) $\left(2\hat{i}+3\hat{j}\right)m/s^2$ **(D)** $\left(\hat{i}-\hat{j}\right)m/s^2$

13. Two charges q_1 and $-q_2$ are initially held at rest at distance r on a smooth horizontal non conducting surface. Then

(A) the centre of mass of the charge system moves towards q_1.

(B) the centre of mass of the charge system moves towards $-q_2$.

(C) the centre of mass of charge system remains at rest.

(D) data insufficient.

14. If net external force on a system of particle is zero.

(A) The centre of mass of the system must be stationary.

(B) The centre of mass of the system must be in motion.

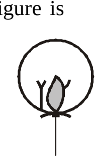
(C) The momentum of the system must be constant.

(D) THe momentum of system must be zero.

15. Ram of mass 80 kg and Sham of mass 70 kg are sitting on a long plank of mass 50 kg placed on smooth horizontal surface at a separation of 5 m. Find distance moved by the plank when they interchange their positions.

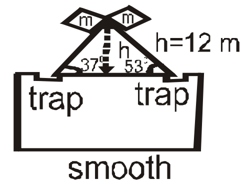
(A) 1 m (B) 0.5 m (C) 0.75 m (D) 0.25 m

16. A bug of mass m is on a balloons of mass M rests in air. A rope of mass m_0 and length L is attached to the balloon. The distance through which balloon moves when bug reaches at the lower end of the rope shown in figure is



(A) $\frac{mL}{m+m_0+m}$ upward (B) $\frac{mL}{m+m_0+m}$ downward

(C) $\frac{mL}{m+m}$ upward (D) L downward

17. 

At the top of smooth wedge frame of mass M = 40 kg are held two small balls of the same mass m = 5 kg (shown in figure). Now balls are released and after some time they get stuck in traps. How far will shift the wedge after the balls are caught in the traps?

(A) 0.7 m in leftward direction (B) 0.7 m in rightward direction

(C) 0.3 m in leftward (D) 0.9 m in leftward direction

18. Two block of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m/s to the heavier block in the direction of the lighter block. The velocity of the centre of mass is

(A) 30 m/s (B) 20 m/s (C) 10 m/s (D) 5 m/s

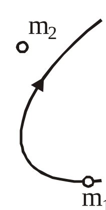
19. A block of mass 4 kg rests on a smooth horizontal surface. It explodes into three fragments due to internal forces. Two of the fragments each of mass 1 kg are found to move each speed $10\sqrt{2}m/s$ in mutually perpendicular direction. The speed of the third fragment is

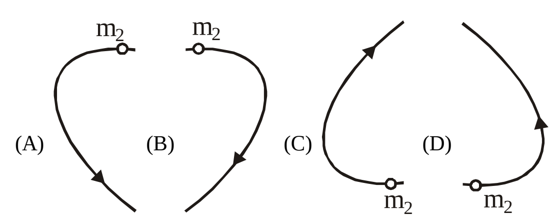
(A) $5\sqrt{2}m/s$ (B) $10\sqrt{2}m/s$ (C) 10 m/s (D) 20 m/s

20. A man of mass 45 kg is sitting on a smooth horizontal surface. He projects a ball of mass 5 kg at a speed of 10 m/s with respect to himself at an angle of $37°$ above the horizontal. Find the recoil speed of the man.

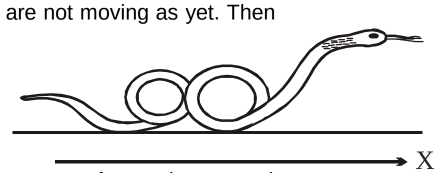
(A) 1 m/s (B) 0.5 m/s (C) 5 m/s (D) 0.8 m/s

21. Two interacting particles form a closed system whose centre of mass is at rest. Fig. illustrates the positions of the both particles at a certain moment and the trajectory of the particle of mass m_1. Select the trajectory of the particle of mass m_2 if $m_2 = m_1$.





22. A snake has started motion in front part of the body while back parts are not moving as yet. Then



(A) Its centre of mass is not moving

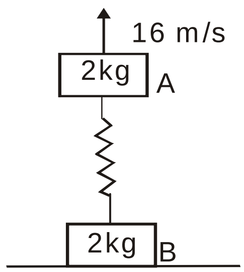
23. Two particle of masses m_1 and m_2 in projectile motion have velocities $\vec{v}_1$ and $\vec{v}_2$ respectively at time t = 0. They collide at time t_0. Their velocities becomes $\vec{v}'_1$ and $\vec{v}'_2$ at time $2t_0$ while still moving in air. The value of

$\left[\left(m_1\vec{v}'_1+m_2\vec{v}'_2\right)-\left(m_1\vec{v}_1+m_2\vec{v}_2\right)\right]$ is

(A) zero (B) $(m_1+m_2)gt_0$

(C) $2(m_1+m_2)gt_0$ **(D)** $\frac{1}{2}(m_1+m_2)gt_0$

24. Figure shows two blocks A and B each of mass 2 kg connected with a light spring of spring constant 100 N/m. The block A is moving ipward with velocity 16 m/s at the instant when block B leaves the surface. How high does centre of mass of the two blocks system rise?



(A) 0.2 m (B) 1.6 m (C) 3.2 m (D) 6.4 m

25. An ice cube of mass m and side length ℓ is placed on a large smooth horizontal fixed thin plate. The acceleration of centre of mass of ice is constant during melting of ice. The time taken by ice to melt is t_0. Find acceleration of centre of mass of ice cube during melting.

(A) $\frac{\ell}{t_0^2}$ (B) $\frac{\ell}{2t_0^2}$ (C) $\frac{\ell}{4t_0^2}$ (D) $\frac{\ell}{5t_0^2}$

26. In previous problem, the normal reaction exerted by the plate during meting is

(A) mg **(B)** $\frac{m}{2}g$

(C) $m\left(g-\frac{\ell}{t_0^2}\right)$ **(D)** $m\left(g-\frac{\ell}{2t_0^2}\right)$

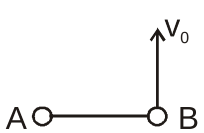
27. In previous problem, the mass of plate is five times the mass of ice cube. If the ice melts, how far does the centre of mass of ice-plate system come down ?

(A) $\frac{\ell}{12}$ (B) $\frac{\ell}{6}$ (C) $\frac{\ell}{3}$ (D) $\frac{2\ell}{3}$

28. A system consists of n particle. Total mass of the system is M = 50 kg. In C-frame, the pseudo force on one of the particle of mass m = 5 kg is $\left(2\hat{i}-2\hat{j}+\hat{k}\right)N$. Find the magnitude of net external force on the system.

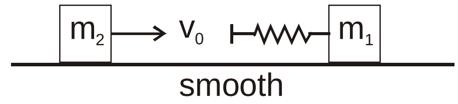
(A) 10 N (B) 20 N (C) 30 N (D) 40 N

29. Two particle A and B having masses m and 2m connected with a light string of length ℓ are placed on a smooth horizontal surface. The particle B is imparted with speed v_0 perpendicular to the string (shown in figure). Find tension in string during subsequent motion.



(A) $\frac{mv_0^2}{\ell}$ **(B)** $\frac{2mv_0^2}{\ell}$ **(C)** $\frac{2mv_0^2}{3\ell}$ **(D)** $\frac{3mv_0^2}{2\ell}$

30.



A block of mass m_2 is moving towards block of mass m_1 (shown in figure). Find maximum compression in spring.

(A) $v_0\sqrt{\frac{m_1m_2}{(m_1+m_2)k}}$ **(B)** $2v_0\sqrt{\frac{m_1m_2}{(m_1+m_2)k}}$

(C) $3v_0\sqrt{\frac{m_1m_2}{(m_1+m_2)k}}$ **(D)** none

One or More than One Options Correct Type

31. A system consists of three particle of masses 1 kg, 2 kg and 3 kg at position (0, 0, 0), (1, 2, 3)m, and (3, 4, 5)m. At t = 0, masses 1 kg, 2 kg and 3 kg are being acted upon by forces $\left(\hat{i}+\hat{j}+\hat{k}\right)N$, $\left(3\hat{i}+4\hat{j}+5\hat{k}\right)N$ and $\left(4\hat{i}-2\hat{j}-\hat{k}\right)N$ respectively.

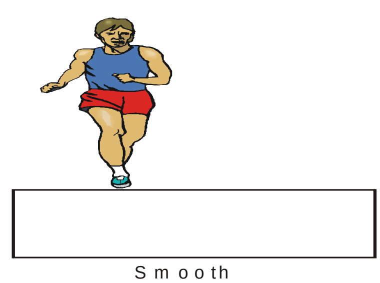
(A) The acceleration of centre of mass of the system is $\left(\frac{4}{3}\hat{i}+\frac{1}{2}\hat{j}+\frac{5}{6}\hat{k}\right)$

(B) **T**he velocity of centre of mass of the system at t = 1 s is $\left(\frac{4}{3}\hat{i}+\frac{1}{2}\hat{j}+\frac{5}{6}\hat{k}\right)m/s$

(C) **T**he momentum of system at t = 1sec is $\left(8\hat{i}+3\hat{j}+5\hat{k}\right)m/s$

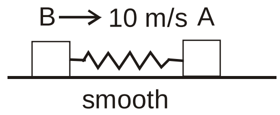
(D) The position of centre of mass at t = 2 sec is $\left(4.5\hat{i}+3.67\hat{j}+1.83\hat{k}\right)m$

32. A man of mass m_0 is standing on a long plank of mass M. Man starts to run on plank with velocity u_0 with respect to plank.



(A) During running, the velocity of centre of mass is zero
(B) During running, workdone by static friction between plank and man is zero.
(C) During running, internal workdone in the body of man is positive.
(D) Total energy of plank and man increases.

33. Two block A and B of masses 1 kg and 4 kg rest on a smooth horizontal surface. They are connected by a light spring of spring constant 100 N/m.



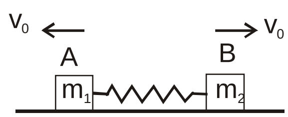
An external kick provides a velocity of 10 m/s to the block B towards the block A.

(A) The velocity of the centre of mass of the two blocks system is 8 m/s

(B) The maximum comprision in spring is $\frac{2}{\sqrt{5}}m$.

(C) The momentum of the block A is 8 kg m/s towards B in C-frame just after external kick.

34. Two block A and B of masses m_1 and m_2 $\left(m_2 > m_1\right)$ respectively are placed on a smooth horizontal surface. The are connected with a light spring constant K and natural length ℓ_0. At t = 0 the two blocks are given velocities as shown in figure. At t = 0, spring is at its natural length.



(A) The velocity of centre of mass of two blocks system at an instant t is $\left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_0$.

(B) At the time of maximum and minimum separation between blocks, the kinetic energy of the two blocks system in C-frame is zero.

(C) The minimum separation between block is $\ell_0 - 2v_0\sqrt{\frac{m_1 m_2}{(m_1+m_2)K}}$

(D) The maximum separation between block is $\ell_0 + 2v_0\sqrt{\frac{m_1 m_2}{(m_1+m_2)K}}$

35. 

Two block of masses m_1 and m_2 are connected by a light spring of force constant K and natural length ℓ_0. They are placed on a smooth horizontal surface. At t = 0, a constant force F is applied on the block of mass m_1, when spring is at its natural length.3

(A) The acceleration of centre of mass at an instant t is $a_{cm} = \frac{F}{m_1 + m_2}$.

(B) At t = 0, the acceleration of m_1 is maximum and that of mass m_2 is zero.

(C) At t = 0, accelerations of blocks A and B are zero.

(D) At t = 0, the centre of mass of two blocks system is at rest.

36. In previous problem,

(A) The work done by pseudo force on the system in C-frame is zero.

(B) The minimum separation between block is ℓ_0.

(C) The minimum separation block is $\ell_0 - \frac{2m_2 F}{(m_1+m_2)K}$.

(D) The minimum separation between block is $\ell_0 + \frac{2m_2 F}{(m_1+m_2)K}$.

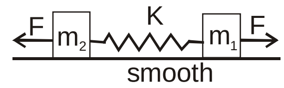
37. 

Figure shows two blocks of masses m_1 and m_2 are connected by a light spring of force constant K_A and natural length ℓ_0. At t = 0, te spring is in its natural and block are released from rest. If blocks are on a system,

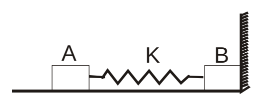
(A) The C-frame of the sustem is inertial in nature.

(B) The centre of mass of the system will remain fixed during subsequent motion.

(C) The minimum separation between blocks is ℓ_0.

(D) The maximum separation between blocks is $\ell_0 + \frac{2F}{k}$.

38. Two blocks A and B each of mass m are connected by a light spring of forcew constant k rest on smooth horizontal surface. The block A shifted towards right a distance x_0 and then released at t = 0. At instant $t = t_0$, te block B leaves the wall. Consider blocks A and B as a system.



(A) For 0 < t < t_0, the acceleration of centre of mass is non-zero.

(B) The centre of mass of system will remain fixed during subsequent motion.

(C) The velocity of centre of mass at $t > t_0$ is $= \frac{1}{2}\sqrt{\frac{k}{m}}x_0$

(D) The shifting in centre of mass in time interval t = 0 to $t = t_0$ is $\frac{x_0}{2}$

Comprehension Type

Passage - 1

A3 kg bomb moving in gravity free space explodes in three fragments of equal masses. After explosion the fragments move apart under influence of their long range mutual interaction of conservayive attractive forces. They are moving with velocitys $-2\hat{i}\, m/s, (3\hat{i} + 4\hat{j})m/s$ and $(4\hat{i} - 8\hat{j})m/s$ just after explosion.

39. The velocity of the bomb before explosion is

(A) $\left(\frac{5}{3}\hat{i} + \frac{4}{3}\hat{j}\right)m/s$ (B) $\left(\frac{5}{3}\hat{i} - \frac{4}{3}\hat{j}\right)m/s$

(C) $\left(\hat{i} + \hat{j}\right)m/s$ (D) $\left(2\hat{i} - \hat{j}\right)m/s$

40. If 50 % of the energy released in explosion is converted into kinetic energy of the three fragment. How much energy has been released in the explosiom ?

(A) 100 J (B) 95.34 J (C) 80.4 J (D) 47.67 J

41. After long time, when the mutual interaction ceases to act, the minimum kinetic energy of all the fragments is

(A) 6.83 J (B) 47.67 J (C) 95.34 J (D) 80.40 J

Passage - 2

Two blocks of masses $m_1 = 1$ kg and $m_2 = 2$ kg are connected by a light spring of spring constant 100 N/m. They are at rest on a frictionless surface with the spring at its natural length. A horizontal force F = 6N is applied to the block of mass m_1 for a certain time Δt in which m_1 moves a displacement $\Delta s_1 = 0.1$ m and m_2 moves a displacement. $\Delta s_2 = 0.05$ m. At the end of time

Δt , the kinetic energy of the system in C-frame is 0.1 J.

42. The value of Δt is

(A) $\frac{1}{\sqrt{3}}s$ (B) $\frac{1}{\sqrt{15}}s$ (C) $\frac{1}{\sqrt{5}}s$ (D) $\frac{1}{\sqrt{17}}s$

43. The kinetic the energy of centre of mass at the end of the time Δt is

(A) 0.1 J (B) 0.2 J (C) 0.3 J (D) 0.4 J

34. The energy stored in the system at the end of time Δt is

(A) 0.1 J (B) 0.2 J (C) 0.3 J (D) 0.6 J

Statement Type

Direction : This section is based on Statement I and Statement II. Select the correct answer from the codes given below.

A. Both Statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct and Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect

45. Statement - I A car cannot accelerate on a smooth horizontal surface.
Statement -II As road is smooth, net external force on the car along the road is zero.

46. Statement - I There are 50 men in a stationary railway compartment on a smooth track. A physical fight starts between the passengers over some difference of option. Due to this some passengers shift in fornt portion of the compartment. The compartment moves in backward direction.
Statement - II If compartment and the passengers are taken as a system. The centre of mass of the system will remain fixed.

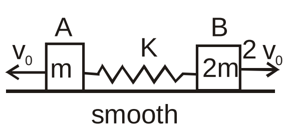
47. Statement-I A sail boat can not be propelled by air blow at the sails from a fan attached to the boat.
Statement -II If sails plus boat are taken as a system the net external force on the system in horizontal direction is zero.

48. Statement - I The kinetic energy of a system of particle is minimum from centre of mass frame of reference.
Statement - II The total momentum of a system as seen from centre of mass frame is minimum.

(A) (B) (C) (D)

Match the Column

49. Two block A and B of masses m and 2m respectivelyare attached to the ends of an undeformed, light spring of force constant K. They are free to move on a smooth horizontal surface. Initially A and B have velocities v_0 towards left and v_0 towards right respectively (shown in figure). A constant force F starts to act on the block B in rightward direction.

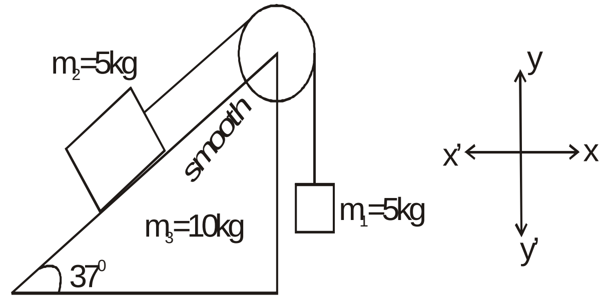


Taking v_0 = 5 m/s, F = 15 N, k = 100 N/m and m = 5 kg, match List I with List II.

	List -I		List II
(A)	The velocity ogf centre of mass (in m/s)of two block system at t = 1s.	(P)	1.96
(B)	The work done by pseudo force (in joule) in C-frame during time interval t = 0 to t = t.	(Q)	0.14
(C)	If the natural length of spring is 1m, the minimum separation (in m) between blocks.	(R)	zero
(D)	If the natural length of spring is 1m, find maximum separation (in m) between blocks	(S)	6

One Integer Value Correct Type

50.

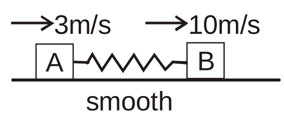


In the situation shown in Figure, the magnitude of acceleration of centre of mass system $(m_1 + m_2 + m_3)$ is $\frac{\sqrt{17}}{n} m/s^2$. Find the value of n.

51. During shooting of super hit film, Rangeela, Amir khan of mass 80 kg and Urmila Matondoker of mass $m_0 = \frac{180}{n}$ kg are enjoying in a motor boat of mass 30 kg. When the boat is at rest on smooth water surface, they are 5m apart and symmetrically located with respect to the centre of boat. Now they come at centre to greet each other. During this, the boat move 1 m towards the bank near to Amir Khan.

52. Consider two fallingballs A and B of masses 1 kg and 4 kg respectively. At t = 0, balls A is moving horizontally with speed 3 m/s and ball B is moving vertically downward with speed 4 m/s.The speed of centre of mass of two balls system at t = 0.5 sec is $\frac{13\sqrt{10}}{n}$ m/s. Find the value of n.

53.



Two block A and B of masses 5 kg and 2 kg attached to the ends of a spring of force constant 1120 N/m are placed on a smooth horizontal surface with the spring in its natural length. Simultaneously velocities of 3 m/s and 10 m/s along the line of the spring in the same direction are imparted to block A and B (shown in figure). The maximum elongation in spring is 5n cm. Find the value of n.

54. A light rope thrown over a smooth and light pulley has a monkey of mass m at one of its end and a counter balancing mass m at other end. The monkey climbs up by a distance $\ell_0 = 10$ cm with respect to rope. Find the magnitude of displacement of centre of mass in cm.

55. 2 small balls having the same mass & charge & located on the same vertical at heights 8m & 10m are thrown in the same direction along the horizontal at the same velocity 10m/s. The height of the 2^{nd} ball is H at instant when first ball strikes on ground at horizontal distance 20 m. The air drag & the charges induced should be neglected. Find the value h in metre.

ANSWER KEY

1. No.

2. $\because \quad p = MV_{cm} \quad \frac{dp}{dt} = \frac{Mdv_{cm}}{dt} = Ma_{cm}$

3. Yes

4. Away from the dog

5. If projectile explodes in air in a number of fragment, the acceleration of the centre of mass is same before explosion and explosion. The cause behind this is that only external force on the system is force of gravity. Hence, the centre of mass continues on the same parabolic path as the projectile's path before exploding.

6. Ballet dancers are famous for graceful jumps in which they seem to hang in the air, momentarily defying gravity. Actually centre of mass of dancer must follow parabolic path. The dancer raises both legs, thus raising her centre of gravity on the way up. As she starts to descend, she quickly lowers her torso. Because she has moved her centre of mass up and down by moving her legs only, her head moves in a nearly horizontal path instead of a parabolic path. As we normally watch the dancer's head, she appears floated horizontally rather than following a parabolic path.

7. Centre of mass of a high jumper travels in a parabolic path once he leaves the ground. As the high jumper goes the bar the body is arched. The centre of mass doesn't have to go over the bar.

8. Zero

9. Yes

10	11	12	13	14	15	16	17	18	19	20
A	A	A	C	C	D	A	B	C	C	D
21	22	23	24	25	26	27	28	29	30	31
B	B	C	C	A	C	A	C	C	A	ABCD
32	33	34	35	36	37	38	39	40	41	42
ABC	ABCD	ABCD	ABD	ABD	ABCD	ACD	B	B	A	B
43	44	45	46	47	48					
D	D	A	A	A	B					

49. $(A \rightarrow S), (B \rightarrow R), (C \rightarrow Q), (D \rightarrow P)$

50. **[9.22]** **51.** **[7]** **52.** **[5]** **53.** **[5]** **54.** **[5]** **55.** **[8]**